# Bayesian Synthetic Control

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# Causal inference

**Causal Effects**

**Experiments**

Randomization allows for exogeneity

**Observational Data**

Need to fix endogeneity arising from lack of randomization

- Differences in Differences
- Instrumental Variables
- Gaussian Copula
- Matching
- Synthetic Controls
- Regression Discontinuity

# What is a synthetic control?

- Technique using pre & post panel data to compare a treated unit to a control constructed as a weighted average of outcomes for untreated units

- Weights are chosen to minimize the differences in pre-treatment outcomes between the treated unit and the synthetic control
    - Parallel trends assumption satisfied by construction

# What is a synthetic control?

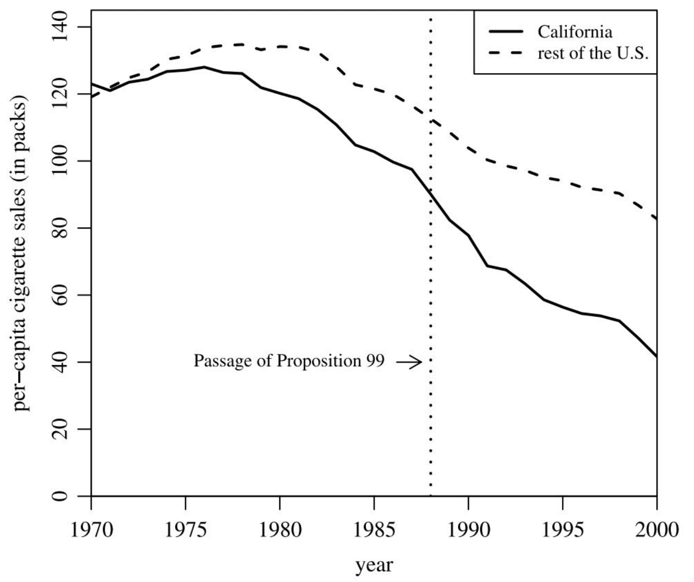



Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

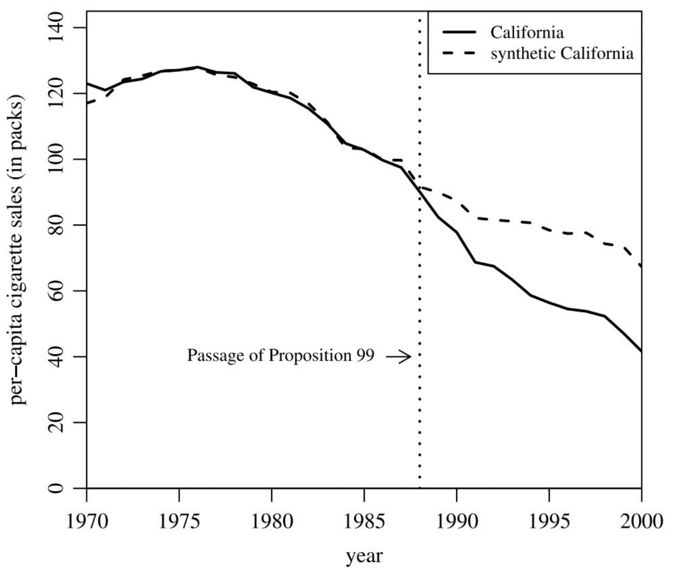



Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

# When should we use a synthetic control?

- Policy change occurs for a single firm/geographic location
- When parallel trends DID assumption cannot be met → no good control unit exists for the treated unit
- **SC Assumptions:**
    - Treated and untreated units are similar prior to treatment (graph)
    - No other events coincide with treatment that impact treated/control differently (theoretical justification)
    - SUTVA: control units are unaffected by treated units (theoretical justification)

# Bayesian vs Standard SC

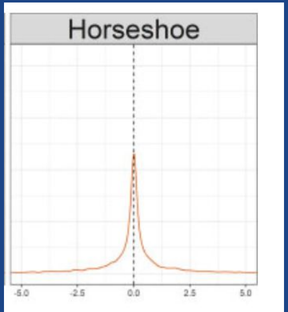
**Synthetic Control:**

- Inference is difficult with only two time series (placebo tests are used with control units)
- Large p, small n & sparsity problems
- Weights are restricted to be positive and between (0,1)

**Bayesian Synthetic Control:**

- MCMC provides credible intervals for inference
- Shrinkage priors help with large p small n and sparsity problems
- Weight constraints are not needed

# Horseshoe BSC Model




$$\beta_j|\lambda_j \sim \text{Normal}\left(0, \lambda_j^2\right) \text{ for } j = 1, \ldots, J,$$

$$\lambda_j|\tau \sim \text{Cauchy}^+(0, \tau),$$

$$\tau|\sigma \sim \text{Cauchy}^+(0, \sigma),$$

$$\sigma \sim \text{Cauchy}^+(0, 10).$$

$$\hat{\beta} = \arg\min_{\beta \in \Lambda} \sum_{t=1}^{T_0} \left( Y_{0t} - \beta_0 - \sum_{j=1}^{J} \beta_j Y_{jt} \right)^2$$

$$\hat{\alpha}_{0t} = Y_{0t}^{I} - \sum_{j=1}^{J} \hat{\beta}_j Y_{jt}.$$

Kim et al (2020)

# Example: Proposition 99 CA (1988)

- Abadie et al 2010
- https://search.r-project.org/CRAN/refmans/tidysynth/html/smoking.html
- CA and 38 control states
- 1970-2000
- Cigarette pack sales per 100,000 people

# Stan Code

# B Stan Codes

## B.1 BSCM-Horseshoe

```
data{
    int N_train; //Number of observations in the pre-treatment periods
    int N_test; //Number of observations in the post-treatment periods
    int p; //Number of control units
    real y_train[N_train]; //Treated unit in the pre-treatment periods
    matrix[N_train, p] X_train; //Control unit matrix in the pre-treatment
    matrix[N_test, p] X_test; //Control unit matrix in the post-treatment
}
parameters{
    real beta_0; //Intercept
    real<lower=0> sigma2; //Error term variance
    vector[p] beta_raw; //Control unit weights (will be transformed)
    //Hyperparameters prior
    vector<lower=0, upper=pi()/2>[p] lambda_unif;
    real<lower=0> tau; //Global shrinkage
```

https://github.com/morganbale/bscm_example

# Workshop

- Download stan code and markdown file from [Github repository]

# Additional ways to learn

- Recreate simulation studies from Kim et al (2020)
- Compare our simulation study to another R synthetic control package
- Compare weights from Abadie et al (2010) to the weights we found using BSCM
- Draw samples from priors and plot (compare web vs paper versions)
- Working with other covariates

# Recap

- Synthetic controls allow us to better estimate causal effects when we don't have an appropriate treatment group to meet assumptions for other causal inference methods (e.g. DID)
- Bayesian synthetic control methods improve upon standard SC by:
    - Giving method for inference automatically through Bayesian sampling
    - Solving the large p, small N problem through shrinkage priors
    - Removing the need for the sum of the weights to be constrained between (0,1)
- Packages available:
    - Synth: R, Stata, MATLAB

# Synthetic Control Papers

- Abadie et al 2010
- Chesnes et al 2017
- Pattabhiramaiah et al 2019
- Wang et al 2019
- Li 2020
- Kim et al 2020
- Abadie 2021
- Kim et al 2020

# Q&A about PhD life

# References

Alberto Abadie, Alexis Diamond & Jens Hainmueller (2010) Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California's Tobacco Control Program, *Journal of the American Statistical Association*, 105:490, 493-505, DOI: 10.1198/jasa.2009.ap08746

Kim, Sunjin, Lee, Clarence, and Gupta, Sachin (2020) "Bayesian Synthetic Control Methods," *Journal of Marketing Research*: 1-22

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